WEB SAG AND THE EFFECT OF CAMBER ON STEERING

by

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ABSTRACT

A web in a horizontal span will deflect downwards due to gravity. This sag is more pronounced for heavy materials in long spans under low tension. It is possible for a web whose unstretched length is greater than the span to be under positive tension.

A cambered web will sag more on the baggy side, especially under low web tension. This side therefore has a longer path length between rollers in good alignment, which provides a mechanism for lateral steering. The boundary conditions of normal entry and velocity matching on the downstream roller (which successfully predict the steering of straight webs by misaligned and tapered rollers) can still be applied without modification.

A horizontal span of cambered web under tension has been simulated using Finite Element Analysis, showing that the web steers to the tight side. The amount of movement varies with span length, tension, density, amount of camber and width. For low tensions, the displacement can be quite large, but it falls rapidly as tension is increased.

NOMENCLATURE

a	Length parameter of catenary curve, arc radius of sagging web
b	Web width
D	Sag at the centre of a span
E	Young's modulus of elasticity for the web
F	Force on cable supports (in Newtons)
g	Acceleration due to gravity, 9.81 ms^{-2}
h	Web thickness
L	Span between supports or rollers
L_0	Unstretched length of web between rollers
MD	Machine Direction
r	Natural radius of curvature of cambered web
s	Arc length along the catenary curve
T	Tension per unit width in web (in N/m) F/b

T_c	Critical tension: just sufficient to straighten cambered web with zero stress on the long side
w	Weight per unit length
x	Horizontal coordinate
y	Lateral coordinate or displacement of web from straight line
z	Vertical coordinate or displacement of web
α	Dimensionless parameter $\rho gL/E$
ΔL	Length difference between long and short edges of cambered web
ε	Tensile strain of web in the span, T/Eh
ρ	Density of web material
σ	Weight per unit area of web ρgh

INTRODUCTION

The effect of gravity on web handling has been described as "nearly negligible". The most common effect of concern is the deflection of rollers under their own weight [1,2]. This can cause misalignment and steering problems for cantilever-mounted rollers, and gathering and wrinkling on wide rollers. It is also acknowledged that the weight of the web can cause it to sag, but this is not normally viewed as a problem. This paper reviews the phenomenon of sag, and then demonstrates that it may cause cambered webs to steer.

The variation of sag across the width of the web may be due to tension changes induced by web steering as a result of the profile and alignment of the downstream roller, whether desired or not. However, some webs show differences in sag even between perfectly aligned, cylindrical rollers. The areas hanging lower are termed *baggy lanes* and are associated with bands of excess length, persisting over many meters [3].

One of the most common, and troublesome, types of baggy web is where the sag increases from one edge to the other. This is caused by an underlying gradient of excess length. When laid out flat, the web will form an arc of a very large circle, with *natural radius r* of several hundred meters or larger (figure 1a). This defect is termed *camber*, *curvature* or *skew*, and may be generated by initial manufacture of the web, or by permanent set during storage on a reel with a diameter taper, in turn caused by a web thickness taper. If the curved web is straightened out, a linear variation across the width of stress in the machine direction (MD) is set up. If sufficient force is applied to just make the longer edge taut (so that it does not curve or wrinkle, but carries zero stress), the average tension can be considered a *critical value* T_c [4], given by:

$$T_c = \frac{Eh\Delta L}{2L_0} = \frac{Ebh}{2r} \qquad \{1\}$$

The web has width b, length L_0, thickness h and Young's modulus E. The length difference is ΔL. The tension varies from zero at the long side to $2T_c$ at the short side, as shown on figure 1b.

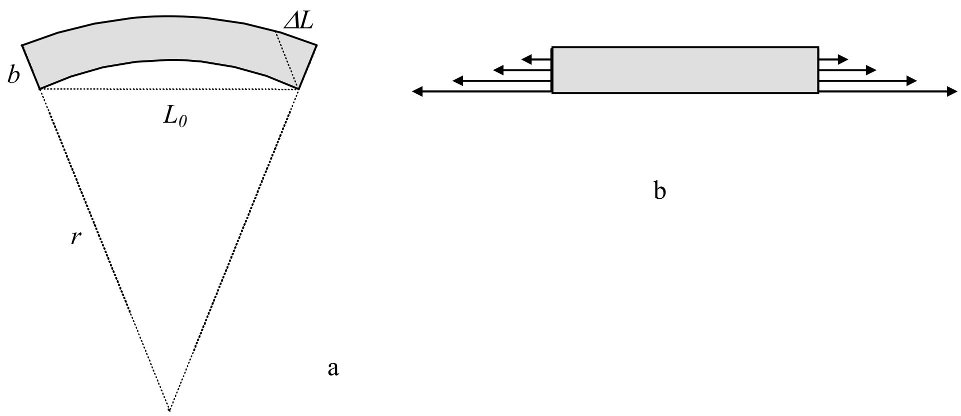


Figure 1 – Behavior of a cambered web
(a) Definition of natural radius of curvature *r*, and link to excess length $\Delta L/L_0$
(b) Distribution of tensile stress when the critical tension is applied to pull the web taut.

A straight web will remain central on a web line consisting of well-aligned, cylindrical rollers. It can be steered towards the side with a shorter path by misaligning a roller, or to the side of larger diameter by introducing a roller radius taper. Both of these effects have been modeled accurately with beam theory [5,6]. Key elements in these models are the boundary conditions at the ends of the beam. Assuming good traction and steady state, the web follows the roller surface velocity at all points after contact. The web strain may change discontinuously as it leaves the upstream roller. However, it must be continuous as it contacts the downstream roller, leading to the condition that the angle of the web there must be parallel to the direction of roller surface velocity, or normal to the axis (the *Normal Entry Rule*). Furthermore, the curvature of the web just before the downstream roller must equal the curvature it must adopt to wrap the roller: if the roller is cylindrical, the web wrapping it must be straight and so the incoming curvature is zero (the *fourth boundary condition*).

Cambered webs often cause problems on long web lines as the downstream end can be displaced by several centimeters from the centre line. A straw poll of delegates at the 2003 International Web Handling Conference did not reach a consensus on the direction of displacement.

The mechanism of steering a cambered web has been considered by several authors. Shelton [4] concluded that a cambered web does not in itself create a lateral error, but it may be steered by a small amount, either by an induced diameter taper effect from the tension gradient causing deformation of compliant surface layers on the roller and web, or by rollers tilted by the asymmetric forces acting on an insufficiently stiff machine. These ideas could account for some of the observations presented in his paper. Swanson [7] measured the lateral displacements of specially made cambered webs: they were larger than Shelton's theory could reasonably predict. He suggested that a modification of the curvature boundary condition was needed, but could not account for the value required. In a series of papers [8,9,10], Olsen generalized the beam analysis to include variations in modulus and thickness as well as excess length, and incorporated shear and sub-critical tension effects. In order to generate steering, he assumed that the web took up its natural curvature at the downstream roller. Brown [11] showed that steering of a cambered web would be a small effect, using both a finite element model and a beam model in which the web took a small but non-zero curvature at the downstream roller.

In order to predict steering of cambered webs between parallel, cylindrical rollers, these authors have abandoned the boundary condition on curvature at downstream roller, which has been successful in predicting steering of straight webs. This paper shows that steering of cambered web arises naturally when sag under gravity is considered, and the correct fourth boundary condition can be preserved.

SAG OF UNIFORM WEB

The Catenary Curve

The shape adopted by an inextensible cable suspended between two supports is a well-known example of the catenary curve (Figure 2). If the two supports are at the same height, the displacement *z* at position *x* measured from one end is:

$$z = a\left(\cosh\left((x - L/2)/a\right) - \cosh\left(L/2a\right)\right) \qquad \{2\}$$

The separation of the supports is *L*, and the length parameter *a* characterizes the curve. The displacement is negative, i.e. downwards. As the tension is raised, the arc length decreases, the cable moves towards the horizontal and the length parameter *a* increases.

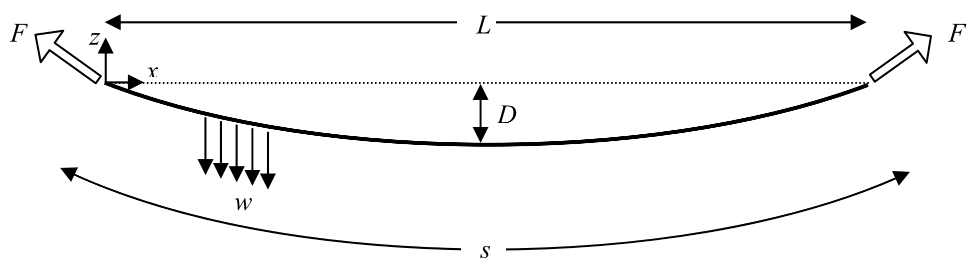


Figure 2 – The Catenary Curve, showing lengths and forces defined in the text.

Unless slack loops are deliberately used as accumulators or length controllers, sag in web lines is usually small. In this case, the hyperbolic functions in equation 2 can be replaced by their small value limits. The shape of the hanging cable is now a circular arc of radius *a*, to a good approximation:

$$z = \frac{x(x - L)}{2a} \qquad \{3\}$$

The *sag* at the centre *D* is:

$$D = \frac{L^2}{8a} \qquad \{4\}$$

and the *arc length s* is equal to:

$$s = L\left(1 + \frac{L^2}{24a^2}\right) \quad \{5\}$$

The force on the supports F balances the chain tension, which changes negligibly along the cable, and is given by:

$$\frac{FL}{a} = ws \quad \{6\}$$

<u>Application to web in horizontal span</u>

These formulae can be modified for an elastic web in a horizontal span between rollers. As the amount of sag increases, the arc of contact on the rollers grows, and the distance between the points of contact decreases. It can be shown that these two effects cancel to a good approximation, so that from the top dead centre points the sag is still given by equation 4, and the total arc length by equation 5. The web is considered to be thin, so that its out-of-plane bending rigidity is negligible, and it would follow a straight horizontal path in the absence of gravity.

The web, thickness h, extends elastically under the lineal tension T, given by

$$T = F / b \quad \{7\}$$

The unstretched length L0 of web is related to the arc length by considering the elastic strain $\varepsilon = T / Eh$:

$$s = L_0(1 + \varepsilon) \quad \{8\}$$

The weight per unit area of the *unstretched* web σ, density ρ is given by:

$$\sigma = \rho g h \quad \{9\}$$

Equation 6 is modified by replacing arc length with unstretched length, giving:

$$\frac{TL}{a} = \sigma L_0 \quad \{10\}$$

These equations can be solved to a first order approximation to give the sag, unstretched length in the span, and vertical radius of curvature:

$$D = \frac{\sigma L^2}{8T} \quad \{11\}$$

$$L_0 = L\left(1 + \frac{\sigma^2 L^2}{24T^2} - \frac{T}{Eh}\right) \quad \{12\}$$

$$a = \frac{T}{\sigma} \tag{13}$$

Finally, they can be rewritten more compactly using a dimensionless parameter α:

$$\alpha = \frac{\rho g L}{E} \tag{14}$$

$$\frac{D}{L} = \frac{\alpha}{8\varepsilon} \tag{15}$$

$$\frac{L_0}{L} = 1 + \frac{\alpha^2}{24\varepsilon^2} - \varepsilon \tag{16}$$

The last two equations are plotted in figures 3 and 4 as functions of strain for various values of the parameter α. The sag is typically a small fraction of the span length, justifying the circular arc approximation. It decreases as web strain (or web tension) increases. Also, it increases as the density increases or modulus decreases, through the parameter α. It is independent of the web thickness if the web strain is kept constant.

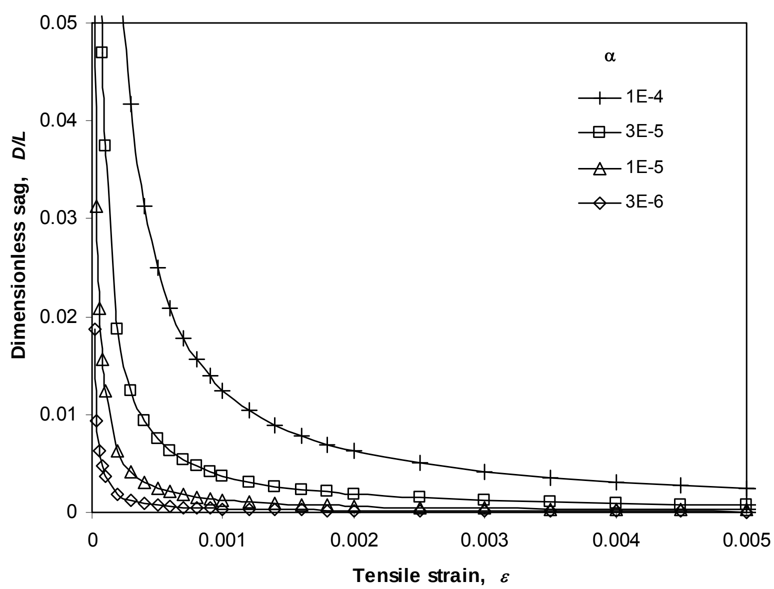


Figure 3 – Variation of dimensionless sag with tensile strain for different values of the dimensionless parameter α.

The unstretched length of web L_0 is longer than the span at low tensions, and shorter at high tension. At high tensions and low values of α, the unstretched length tends towards the value for a straight web path. However, at low tensions the unstretched length increases as tensile strain decreases, and as the parameter α increases from higher density or reduced modulus. Again, thickness has no effect if the strain is kept constant.

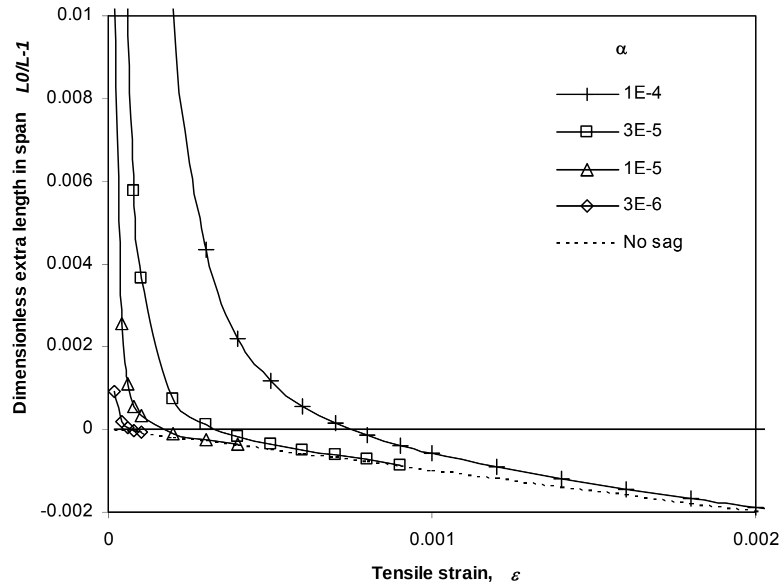


Figure 4 – Variation of dimensionless extra length in the span with tensile strain for different values of α, and in the absence of sag (dotted line).

Typical web behavior

Some typical values for the material properties and the parameter α for a 1 m span are shown in Table 1. Values range from 3.8×10^{-7} for steel and aluminium to 6.3×10^{-5} for low-density polyethylene (LDPE), with paper and stiffer plastics lying between. The metals have less tendency to sag according to the α value, but this may be offset by the lower tensile strain used during transport over rollers. When hot, plastic webs have a much lower modulus, and therefore problems of sag and extra length will be more severe.

Material	Density (kg m^{-3})	Modulus (GPa)	α for a 1m span
Steel	7800	200	3.8×10^{-7}
Aluminum	2710	69	3.8×10^{-7}
Newsprint	780	4	1.9×10^{-6}
PET (biaxially oriented)	1400	4	3.4×10^{-6}
LDPE	900	0.14	6.3×10^{-5}

Table 1 – Values of material properties and the dimensionless parameter α for some web materials and a span of 1 m.

The equations can be replotted in various ways. Figure 5 shows the variation in sag with span length for 100 μm thick PET with modulus 4 GPa at various tensions. There is significant sag for all spans over 5m, and also for shorter spans under the lowest tension. However, the fractional extra length (figure 6) only increases noticeably over the negative value predicted without considering sag at tensions of 200 N/m and below.

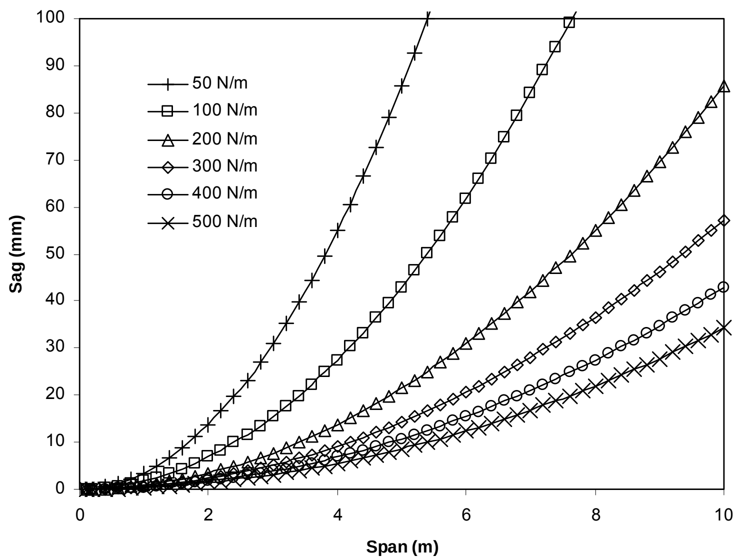


Figure 5 – Calculated sag as a function of span length for 100 μm thick PET under different tensions.

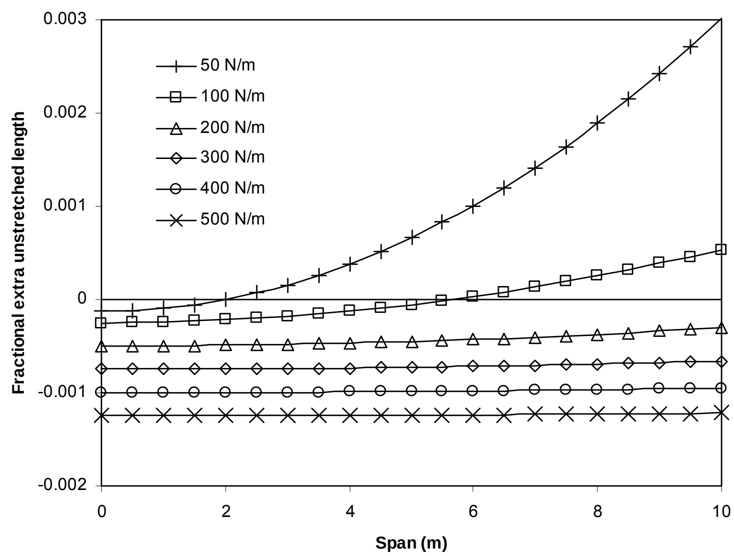


Figure 6 – Predicted variation of fractional extra length with span for 100 μm thick PET under different tensions.

Web Line Implications

Layout. A prediction of web sag under typical conditions allows a web line to be designed so that other components are placed a sufficient distance beneath the nominal web path to avoid undesired contact. Alternatively, any large horizontal spans can be shortened by remounting or adding extra rollers.

Inclined spans. The weight of the material can be resolved into components acting along and perpendicular to the straight web path. In most situations, the component acting along the web produces a very small increase in tension at the upper end, which can safely be ignored. The perpendicular component can be used with the same equations as before, but with a lower effective value of g. In the rare situations of heavy webs, the tension varies along the length and the hyperbolic form of the catenary is needed.

Taking up slack. The unstretched length of web in a line with several long horizontal spans and low tensions may be several centimeters longer than the web length calculated assuming a straight web between rollers. However, simple inspection and touching the web will show there is tension in all the spans. Nevertheless, if the tension is increased by, say, increasing torque to the winder, there will be significant movement as the "slack" is taken up and the amount of sag is reduced.

Steady state tension profile. With good traction on all rollers, the tension profile along a web line is governed by the differences in roller surface speeds [12]. The profile is unaffected by the actual lengths between rollers. Hence web sag has no effect on the steady state tension profile. A similar effect was noted by Ries [13] for the variable web path in an air flotation dryer.

SAG OF CAMBERED WEB

When cambered web travels through a span, the longer side sags more and has both greater arc length and unstretched length. If the curved surface taken up by the web is flattened, its two ends therefore not parallel, and therefore a steering effect similar to that of a misaligned roller is expected. The web should deflect towards the shorter side, and the lateral displacement will be larger for more severe camber, greater sag, longer spans and lower tensions.

However, steering of a straight web by a misaligned roller is accompanied by an in-plane bending moment at the upstream roller, which produces a tension gradient across the web. In the cambered web, this additional moment will oppose the tension gradient at the upstream end. In turn this will reduce the difference in sag between the two edges, and hence the steering effect will be smaller than anticipated.

An analytic solution of this problem has not been attempted: instead the finite element analysis package ABAQUS® has been used to simulate it.

Boundary Conditions

This work is guided by the philosophy that the same principles that work for a straight web should work for a cambered web, as described in the introduction.

At the upstream roller, the lateral position y of the web is fixed. Web shear may be present as the web travels on the roller as a result of steering in the previous span [14]: however, this can be set to zero to characterize the effect of the span under consideration. The shear force set up by steering may lead to a shear strain after the web leaves the roller, and this will cause the centre line of the web to leave the upstream roller at an angle. This is expressed as a relation between the first and third derivatives of the lateral displacement y with respect to x at the upstream end of the span. For long spans, the shear strain is small and can be assumed to be zero, giving a second boundary condition of zero angle as the web leaves the upstream roller.

On the downstream roller, the web approaches the roller parallel to the direction of the surface velocity, so the normal entry rule sets the angle of entry, zero for parallel rollers.

If the downstream roller is cylindrical, the fourth boundary condition is zero web curvature at the end of the span. In a straight web, this implies a zero bending moment, and in a cambered web, a bending moment equal to that required to straighten the web. If the roller has a diameter taper, the web curvature at the end of the span must match that required to wrap the roller: a bending moment is developed for a straight web. Equivalently, the tensile strain in the web must develop a gradient in the lateral direction to match the surface velocity gradient caused by the diameter taper.

Applying these boundary conditions to a cambered web moving in a plane between parallel cylindrical rollers results in the web traveling straight [4]. The MD tensile stress varies linearly across the web, matching the stress required to straighten the web as it wraps the downstream roller.

As 3 dimensions are now being considered, there is the additional condition that both ends are fixed in the vertical direction. As before, the movement of the line of contact around the roller as the sag increases is neglected.

FINITE ELEMENT SIMULATION

Quantity	Base Case	Swanson
Material	PET (typical)	PET
Density	1400 kg m^{-3}	1400 kg m^{-3}
Young's Modulus	4 GPa	4 GPa
Poisson's Ratio	0.3	0.3
Thickness	100 μm	30 μm
Span	5 m	2 m
Width	0.5 m	0.3 m
Mesh size	40 × 8	40 × 8
Tension	100 N/m	73.3, 220 N/m
Acceleration due to gravity	9.81 m s^{-2}	9.81 m s^{-2}
Radius of curvature	2000 m	246 m
Initial stress gradient	-0.5 to 0.5 MPa	-2.44 to 2.44 MPa
Critical Tension	50 N/m	73.3 N/m

Table 2 – Properties used in the finite element simulation for the base case and to reproduce two cases reported in [7].

Model description

The web between the lines of contact on the upstream and downstream rollers was modeled using ABAQUS/Standard version 6.5. A *base case* was selected to have a long span, low tension and sizable camber to enable any steering effect to appear clearly. The span to width ratio of 10 was chosen to minimize the importance of shear strains. The applied tension was sufficiently low not to affect the bending behavior ("*kL*" much less than 1), but was high enough to ensure the tensile stress remained positive at all points.

A second set of parameters was used to simulate two of the experiments reported by Swanson [7]. Not all variables are defined in his paper, and there is an inconsistency between the critical tension and natural radius of curvature. The parameters for the base case and the simulation of Swanson's experiments are shown in Table 2.

The mesh, shown in figure 7, used first-order reduced integration shell elements (S4R). The mesh was produced as rectangular but with a gradient of initial MD tensile stress to introduce camber. Material properties were isotropic, linear elastic. The non-linear solver was used, and each loading step was carried out in a minimum of 10 increments.

Boundary Conditions and Loading

To keep the web fixed in space and define the contact line, the nodes on the upstream end were held fixed in *x* and *z*. The central upstream node was also held fixed in *y*, maintaining the lateral position of the web while allowing the edges to move in and out through Poisson contraction[1].

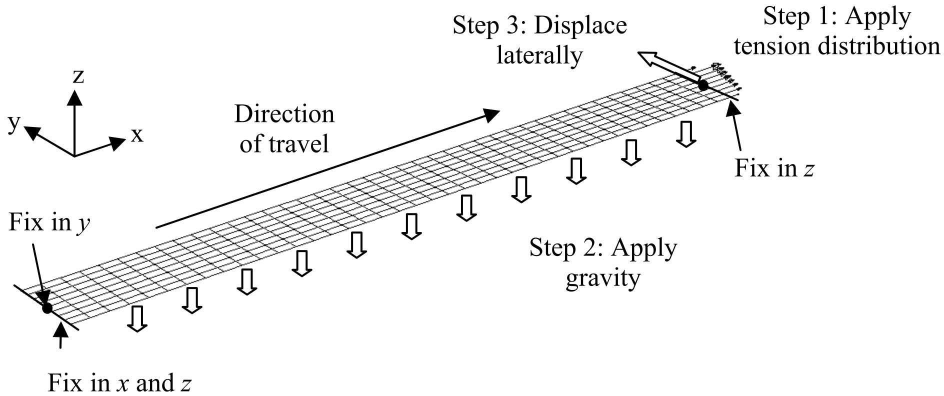


Figure 7 – Mesh for Finite Element simulation, initial boundary conditions sustained through the run and the loading steps.

In an auxiliary run, the downstream end nodes were displaced uniformly in *x* to generate the required tension. The web was then in the state expected for transport under tension between two parallel cylindrical rollers in the absence of gravity. The stretching was accompanied by Poisson contraction, but no lateral movement of the central downstream node. This absence of any steering supports the arguments of Shelton [4] for the straight running of a cambered web. In this state, the reaction forces on the downstream nodes were recorded, as these represent the distribution of tension as the web touches the roller.

In the main run, the concentrated forces recorded in the auxiliary run were applied in step 1 (Figure 7), and allowed to follow the rotation of the nodes resulting from sag and

[1] A more accurate condition may be to hold all nodes on the upstream end in *y* after initial tension has been applied. This change will not significantly affect the results.

lateral displacement. The downstream end nodes were unconstrained in x. The web reached exactly the same state as in the auxiliary run.

In step 2, gravity was applied whilst maintaining the applied forces on the downstream end. As expected, the web developed sag, larger on the side under lower tension, and the downstream end moved towards that side. The MD tensile stress distribution changed very little in this step.

As the downstream end of the web was no longer aligned parallel to the y-axis, step 3 was added to displace the central downstream node in y until the rotation of that node in the x-y plane returned to zero, corresponding to the normal entry rule[2]. At this point, the web satisfied the required boundary conditions, and the lateral displacement of the central node was recorded.

In this loading procedure, the span does not remain constant, but the initial unstretched length of web does. However, this merely means that the steering and sag results are appropriate to the new value of the span, not the initial one. This difference is insignificant for all practical sag and tension values.

Base case results

After gravity was applied in the second step, the web sag was 72 mm at the low tension side and 30 mm at the other, and the downstream end moved 13 mm towards the long side. In the final step, shown in Figure 8, the sag became more even, varying from 52 to 36 mm, and the lateral displacement was 1.643 mm towards the short side.

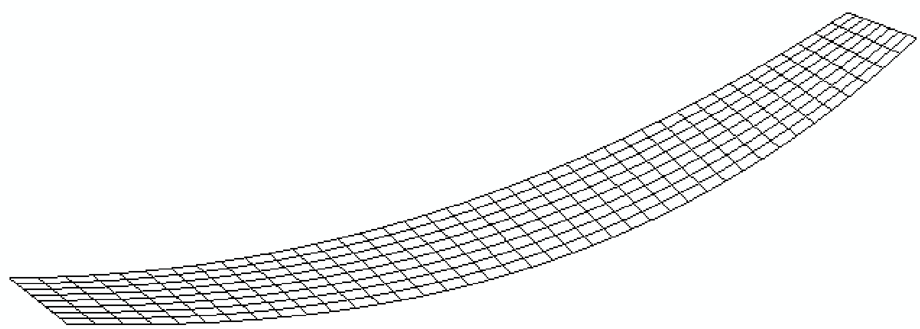


Figure 8 – Deformed shape of web in the base case, with displacements magnified by ×10

The stress distribution at the downstream end remained a linear variation from 0.5 to 1.5 MPa (see Figure 9). However, at the upstream end, the stress was 0.18 MPa higher at the longer side, indicating that not only had the natural curvature developed, but also some additional steering was taking place.

Checks

Several checks were applied to the simulation to ensure it was working correctly:

[2] This is strictly only true in the absence of shear strain.

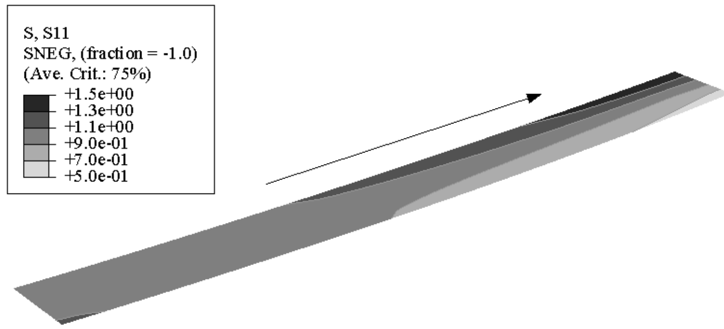


Figure 9 –Contour plot of MD tensile stress superposed on deformed shape of the base case at the end of step 3. Contours range from 0.5 MPa (light) to 1.5 MPa (dark).

Natural Curvature. After applying the initial stresses only, the downstream end was released. The central node was displaced by -0.01 mm in *x* and 6.249 mm in *y*. This is in excellent agreement with predictions. In addition, the maximum stress in the web fell to 10^{-4} MPa, indicating that it had become stress-free.

Steering of straight web by misaligned downstream roller. The web with no camber or gravity acting was deflected 0.33 mm by a rotation by 10^{-4} radians, in exact agreement with the simple beam theory.

Steering of straight web by diameter variation. With no camber or gravity acting but the same downstream distribution of loads, the web moved by 1.93 mm towards the high tension side. This is rather smaller than the 2.08 mm predicted by beam theory.

Sag of straight web. The model showed the same sag as the analytic theory: 42 mm.

Reversing gravity. Changing gravity to act upwards reversed the sag but not the steering direction.

Comparison with the measurements of Swanson

The simulation disagrees with the results of Swanson [7], as shown in Table 3. The steering effect is in the opposite direction, and there is a much stronger effect of tension than Swanson found (see below). Brown [11] also modeled the experiment, and found a very small steering effect but in the same direction as the measurements. He pointed out that conditions in the preceding span may have influenced the experimental data.

Case number	**Tension (N/m)**	**Measured displacement (mm)**	**Model prediction (mm)**
2	220	-0.3	5×10^{-3}
4	73.3	-0.24	0.12

Table 3 – Comparison of Swanson's measurements (selected from table 1 of [7]) and simulation results. Positive displacements are towards the short side.

Effect of varying conditions

The effect of simulation parameters was explored by changing one variable at a time. In no case was there a steering effect during the initial extending of the rectangular mesh to set up the loads, nor did the steering effect change direction once the boundary conditions were satisfied. Combinations of variables have not yet been explored.

Tension. As expected, increasing tension decreases the steering, as shown in Figure 10. Tensions below the critical value of 50 N/m were obtained by setting the loads to zero over part of the width, rather than applying compressive forces to the nodes [10]. This was hoped to simulate the slack edge lifting off the roller slightly [4]. The lowest tension run (28 N/m) suffered from numerical problems, and the downstream end of the web distorted significantly from a straight line, leading to some uncertainty in the displacement. Other runs close to or below critical tension required the downstream end of the web to be fixed in y during step 2 to prevent convergence problems. All other runs worked well.

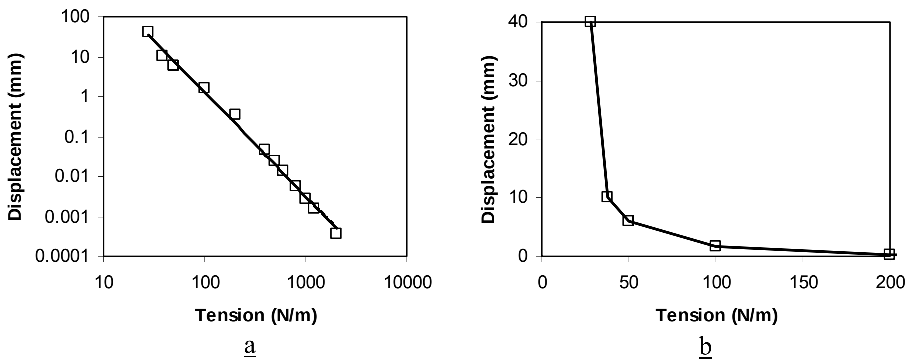
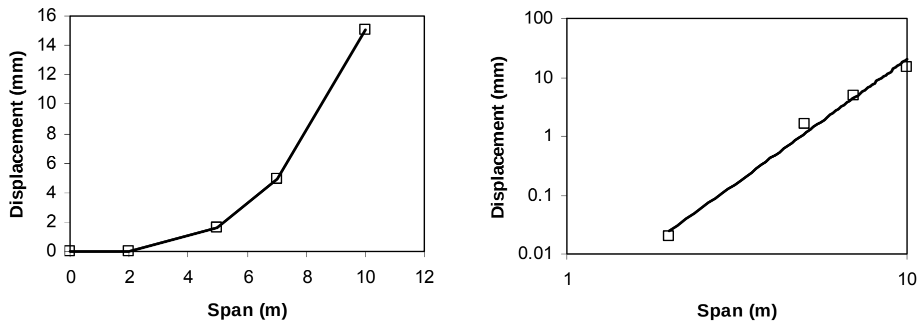


Figure 10 – Effect of tension on steering displacement
(a) Logarithmic plot
(b) Linear plot at low tension

Figure 10a shows a logarithmic plot of the results, with the trend line indicating a strong $T^{-2.6}$ dependence. The displacement at low tension exceeds the 6.25 mm of the web displaying its natural curvature. There does not appear to be a significant change in behavior at the critical tension. This region is expanded on a linear scale in Figure 10b.

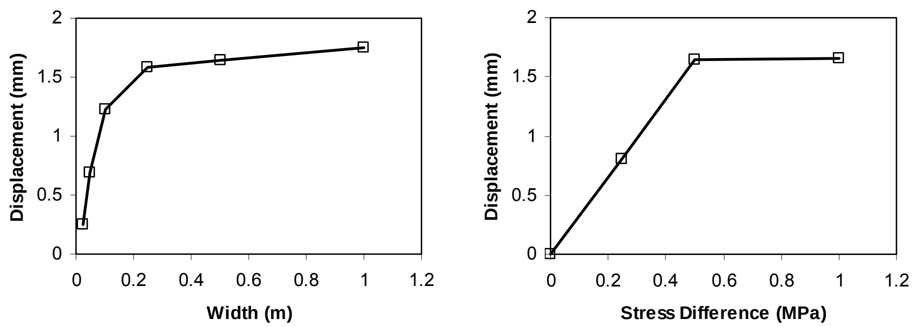
Span. Span also has a strong effect on steering, as shown in Figure 11. The trend line on the logarithmic plot indicates a $L^{1.9}$ dependence. This is close to the L^2 dependence of displacement due to natural curvature. Both span and tension affect the steering through their strong effect on overall web sag.

Width. Figure 12a shows the effect of width at constant tension of 100 N/m and the same web curvature, leading to an increase in stress difference across the web as width increases. The number of nodes in the MD was increased at narrow width to maintain a reasonable aspect ratio for the finite elements. At very narrow widths, the steering effect tends to zero. The displacement increases with web width, but the effect decreases approaching the critical value of 1m, where one edge is nominally under zero stress.



a b

Figure 11 – Effect of span on steering displacement
(a) Linear plot
(b) Logarithmic plot



a b

Figure 12 – Effect on steering displacement of:
(a) Width, from 50 mm to 1 m, keeping the same stress gradient or web curvature
(b) Camber, as initial stress difference between the two edges of a 0.5 m wide web

Camber. Figure 12b shows the effect on steering of camber, specified by the difference in initial stress between the two edges, when the web is flat and straight. Initially, the dependence is linear, but again there is very little extra effect as the critical condition (1 MPa for the constant tension of 100 N/m) is approached.

Density. Figure 13a shows that the steering displacement increases with density with a nearly linear dependence.

Modulus. The runs with different modulus were carried out under the same loads. Therefore a higher modulus run has lower curvature (increased radius). The tension is also unchanged, but the tensile strain will be lower. Figure 13b shows that raising modulus in this way has a small negative effect on steering displacement. If instead, the radius of curvature is kept constant, a combination of the effects shown in figures 10b and 12b will apply, and the displacement may increase with modulus.

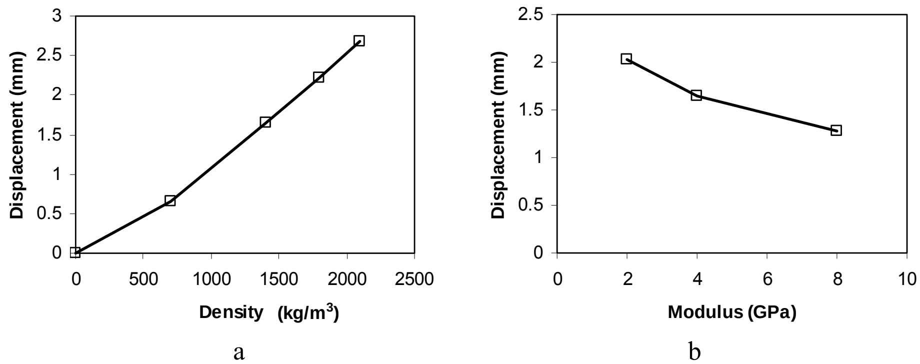


Figure 13 – Effect on steering displacement of:
(a) Density
(b) Modulus, at the same tension and stress difference from camber

Effect of web span orientation

By altering the direction along which gravity acted, the effect of different web orientations could be simulated.

If the web ran vertically, there was no steering displacement and the linear stress distribution across the web was maintained. The stress at the upper end was 66.7 kPa higher than that at the bottom, very close to the 68.7 kPa predicted from the weight of material, and very small in comparison with the tensile stress of 1 MPa.

If the web ran horizontally but the transverse direction was vertical, there was a downward steering effect of 1.80 mm, the same for straight and cambered web in both orientations. Beam theory predicted 1.43 mm, close but not identical to the simulation.

Hence, a component of gravity acting perpendicular to the span is necessary to develop a difference in sag across the width of a cambered web and steer it.

DISCUSSION

This paper has shown that the path length differences across the web can lead to lateral steering. In a model for the similar situation of a cambered web in an air flotation oven, Moretti [15] showed that the web path length was shorter at the high tension side, and the web steered in this direction with full traction on the exit roller[3].

Finite Element Technique

The commercial package "ABAQUS" has the advantage over purpose-written finite element code of many years' development and verification in many different test simulations, leading to confidence in the reliability of its formulation and results. This simulation could be extended to cover different web problems, such as baggy lanes, thickness and modulus variations across the width, material anisotropy and orientation variations, downstream roller angle and diameter variations.

Incorporating the boundary conditions into the finite element model proved somewhat tricky. The normal entry rule and zero curvature mean that 2 conjugate

[3] His model included a steering roller at the exit of the dryer span to bring the web back to zero displacement. A negative steering angle was required, indicating that a parallel downstream roller would have resulted in a web displacement to the high tension side.

variables (rotation and torque) are both specified together at the downstream end: this combination is not permissible in ABAQUS. The method adopted overcomes this issue but is still not perfectly correct: further improvements may allow shear to be included correctly.

Validation

The results of this simulation have not yet been verified. The identification of important variables now allows an experimental program to be put together, in order to explore the predictions. In addition to the lateral displacement, the vertical displacement throughout the span and the shear force on the rollers may prove useful measurements to test the simulation. An analytical formulation using theory for out-of plane deflection of a curved beam may be possible: however, it may not be very accurate because the simulation indicates that the web bends in the width direction.

Steering with poor traction

The simulation has so far covered the case of good traction, which leads to the boundary conditions used. Once traction is lost, the frictional forces determine the position of the web [16]. Neither the normal entry rule nor the zero curvature may be expected to hold. Swanson [7] assumed the normal entry rule would still hold, but the curvature would lie somewhere between zero and the natural value, leading to small steering displacements towards the low tension side.

The intermediate results in step 3 of this simulation show that the direction of steering can reverse to the long side if the downstream curvature remains low but the lateral force is lower than required to maintain normal entry. There is a tempting analogy with steering of a straight web by a tapered diameter roller, where the steering direction changes direction when traction is lost. However, there is insufficient proof from this work for the speculation that this will also apply to a cambered web.

There is no method for determining the frictional loads on the web with poor traction, and the adjacent span at least must also be included in the simulation. A finite element model for this could be developed, but would have to include frictional contact with the rigid rollers and run sufficient web through to set up steady state conditions [12].

Effect of sag on steering of straight webs

To date, the effect of sag on the steering of straight webs has not been considered. A tension gradient across the width is set up by steering, and so a variation in sag is also expected. Again this would be accompanied by web path length differences, and so cause a departure from the beam theory of steering. An experimental study may verify the effect of sag independently of camber, and also be easier to carry out.

CONCLUSION

Web under tension in a horizontal span will sag under its own weight, also increasing the unstretched length of web in the span. A cambered web will sag more, and be under lower tension, at the long edge.

A Finite Element simulation has been used to demonstrate that a cambered web steers towards the high tension side when sag is present and traction is good. The boundary conditions that have been successfully used for straight webs need no modification. This may be the solution to the 30-year old problem of understanding the steering of cambered webs. The direction prediction is opposite to limited published measurements. Experimental verification of this new simulation is required.

The steering effect is large at low tensions, and it may exceed the displacement from the natural curvature. Raising tension has a strong effect and rapidly reduces the steering displacement. Span also has a major effect. The steering increases with web width and with the natural curvature, but both effects appear to saturate. Density has an approximately linear effect.

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Web Sag and the Effect of Camber on Steering **D. P. Jones,** Emral Ltd., UK

Name & Affiliation	Question
Ron Swanson, 3M	That makes me think that if all our web lines went straight up in the air we wouldn't have any problem.
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	I'd say yes.
Name & Affiliation	**Question**
John Shelton, Oklahoma State University	Accumulators in the steel industry are straight up in the air and camber affects the lateral position toward the long side. Accumulators, when they are full, have long spans. Bruce Feiertag has said that a cambered web, in the steel industry, which is the most troublesome industry for lateral behavior, goes toward the long side. Camber is worse in the steel industry because the strain is so low. Bruce has tremendous experience in the steel industry. I sort of regret publishing my 1997 paper on camber because it was wrong. Ron Swanson pointed out from his experiments that it was nowhere near a square relationship between lateral movement toward the long side and span. I agreed with him as soon as he came up with that conclusion. The 1997 report's accomplishment was to get other minds (Jan Erik Olsen, you and Ron and others) to think about cambered webs. I don't think we are at the solution yet.
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	I think I don't have the depth of experience in other industries. I do believe the simulations show that horizontal spans can act as a mechanism for steering. If we just have a vertical span, this model isn't going to give you any steering. I did a run with gravity acting in the plane of the web and there is no steering effect whatsoever.